Stability of Partially Lined Combustors with Distributed Combustion

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An integral method for theoretical evaluation of the stability of confined flows with mass sources is presented. This method is applied to evaluate the effect of partial length acoustic liners on liquid propellant combustors with distributed combustion sources. The combustor is modeled as a right circular cylinder terminated by a "multiorifice" (constant Mach number) nozzle. The spatial spread of combustion in the chamber is represented by an arbitrary number of pressure sensitive planar mass sources. On the cylindrical periphery of the chamber is an acoustic absorber of arbitrary length, position, and damping capability. Integral equations are developed from the partial differential equations describing the nonsteady flow and are solved iteratively. Results are given in terms of linear neutral stability plots using a pressure sensitive combustion response. Calculations indicate that increasing the axial spread of combustion increases the stability of the combustor and the effectiveness of the absorber. The most effective liner placement appears at the location of greatest combustion concentration.

Nomenclature

a	= speed of sound
$G(\mathbf{R}/\mathbf{R}_0)$	= Green's function
$G_N(\mathbf{R}/\mathbf{R}_0)$	= modified Green's function
G	= nozzle response $= 1/\gamma + \beta_{noz}/M_\xi$
h	= enthalpy
$\imath$	= unit imaginary $= (-1)^{1/2}$
i	= ith continuous flow region
j	= positive integer
J_m	= Bessel function of the first kind of order m
K	= liner impedance $= 1/\gamma\beta_c$
l	= positive integer
L	= chamber length
M	= Mach number
m	= positive integer
$\dot{m}$	= mass generation flux
n	= positive integer or interaction index
N	= combustion response
$\mathbf{n}$	= outward unit surface normal vector
P	= pressure
$\mathbf{q}$	= velocity
r	= radial coordinate
R	= radius of chamber
$\mathbf{R}$	= position vector
$\mathbf{R}_0$	= Green's function source position vector
S_{c_i}	= surface area of liner in region i
S_{noz}	= nozzle surface area
S_{inj}	= injector surface area
$S_{\zeta i}$	= surface area of planar source
t	= time
T	= temperature
u	= axial velocity component
u_l	= droplet velocity
V	= volume
X	= liner length
X_1	= length from injector to the beginning of the liner
X_2	= length from the injector to the end of the liner
Z	= axial coordinate
Z_{ζ_i}	= axial coordinate for the ζ_i combustion source
β_{inj}	= acoustic injector admittance $= M_1(1/\gamma - N)$
β_c	= liner acoustic admittance
β_{noz}	= nozzle acoustic admittance
γ	= ratio of the specific heats
η_{lmn}	= acoustic eigenvalue for l, m, n
θ	= circumferential coordinate
λ_{lm}	= roots of $J_m'(\lambda_{lm} r)\|_{r=1} = 0$
$\Lambda_{lmn}^{1/2}$	= normalization constant of the eigenfunctions
μ_{ln}	= coefficient matrix in the velocity potential expansions
ρ	= density
σ	= entropy
τ	= sensitive time lag
ϕ	= velocity potential
ψ	= normalization constant for the potential
Ω_{lmn}	= eigenfunction with wave numbers l, m, n
ω	= complex frequency
ω_0	= reference frequency = 1.84118
ζ_i	= combustion source i
ξ	= total number of continuous flow regions
$\delta n, \hat{n}$	= Kronecker Delta
$\delta(R-R_0)$	= Dirac Delta

Subscripts

c_i	= quantity evaluated on the chamber cylindrical periphery within region i
inj	= injector quantity
i	= quantity pertaining to the ith continuous flow region
noz	= nozzle quantity
ξ	= quantity evaluated at the entrance of the nozzle
ζ_i	= quantity evaluated at the ith combustion source

Superscripts

$\circ$	= stagnation quantity
$'$	= perturbation quantity
$*$	= dimensional quantity
$\underline{j}$	= approximation of order j
$-$	= steady-state value
$\sim$	= chamber quantity with no liner present
$\hat{}$	= designates particular mode integers $(\hat{l}, \hat{m}, \hat{n})$

Introduction

SINCE the 1950's, the unsteady behavior of gaseous flow within combustion chambers has been the subject of extensive analysis by many researchers. Owing to mathematical difficulties inherent in this problem these models have been idealized for special cases. For example, within the realm of high-frequency combustion instability, stability has been studied for combustors with low Mach number flow,[1] combustors with special geo-

Received May 14, 1973; revision received October 9, 1973. This work was sponsored by NASA Grant NGR 06-002-095.
Index categories: Combustion Stability, Ignition and Detonation; Wave Motion and Sloshing.

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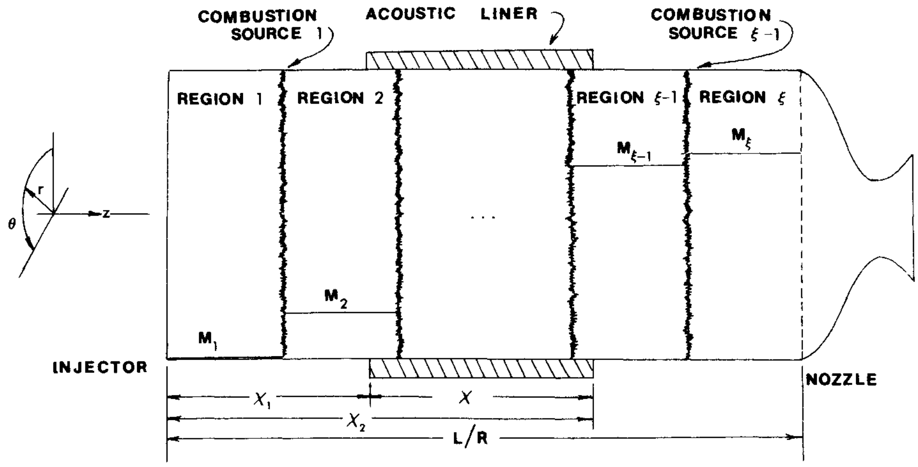

Fig. 1 Combustion chamber model with distributed combustion sources, partial length acoustic liner and "short" nozzle.

metries[2] and combustors with a single concentrated combustion source.[9]

Current interests in mechanical damping devices for combustion instability (acoustic liners, baffles, etc.) has motivated more research and has simultaneously, increased the complexity of the problem. Oberg and Kuluva[6] bypassed some difficulties by studying the stability influences of an acoustic liner on a combustor modeled with no mean flow or nozzle influences. Priem and Rice[7] carried out an analysis including these influences in an approximate way for a combustor with a full length acoustic liner and a concentrated planar source of combustion at the injector face. A more general and accurate integral method was developed by Mitchell, et al.,[9] studying the influences of partial length acoustic absorbers on a comparable combustor model.

Partial length acoustic liner influences on combustors modeled with an arbitrary axial spread of combustion, in contrast with the concentrated combustion or low Mach number mean flow models, haven't been previously treated.§ Therefore, the intention of this work was to develop an analytic model to determine the linear stability behavior of combustion chambers with distributed combustion and partial length liners.

The present study represents the distributed combustion occurring in the combustor through the use of planar flow discontinuities incorporating gaseous mass production. The axial spread of combustion sources and specification of their strengths then defines the combustion distribution being considered. An integral method is then developed and applied to the resulting discontinuous gasdynamic flow equations in order to determine the combustor's stability behavior. Results are given in terms of the combustion response model used to predict the response of the planar sources to pressure perturbations.

Model and Assumptions

The integral method presented herein is applied to evaluate the stability of combustors with distributed combustion sources and partial length acoustic liners. Three-dimensional oscillations are studied in a combustion chamber modeled as a right circular cylinder terminated by a "multiorifice" (constant Mach number) nozzle.[3] The acoustic absorber (i.e., series of Helmholtz resonators) lines the cylindrical periphery of the chamber and is of arbitrary length, axial position and damping capability.

In order to simplify the analysis the combustion process within the chamber is modeled as a vapor generation process which occurs instantaneously at specified locations. The unperturbed gaseous field is, therefore, represented as a series of uniform flow regions with planar combustion sources connecting consecutive regions (Fig. 1). Due to mass addition, the combustion sources, which are distributed axially down the chamber, discontinuously change the character of the flow from region to region.

The gasdynamic field between consecutive combustion sources is assumed to be composed of thermally and calorically perfect combustion gases with no heat-transfer or diffusion processes taking place. (The liquid phase presumably occupies a negligible volume within each region.) Within each region the flow is assumed to be inviscid with no momentum exchange between the gaseous phase and the liquid phase in the form of droplet drag. Consequently, it is consistent to take the flow to be irrotational.

Analysis

A first-order linear perturbation method is used in this analysis which limits the oscillations to small amplitudes. From the linearized system of equations a Green's function method is used to obtain the first-order perturbations of the state variables. This analytical treatment is most appropriate for this problem because of the discontinuous boundary conditions for the gaseous field caused by the acoustic liner and the combustion sources.

Before these methods are applied, the state variables are nondimensionalized by mean steady-state gas conditions at the injector (i.e., $\rho = \rho^*/\bar{\rho}_{inj}^*$, $q = q^*/\bar{a}_{inj}^*$, etc.). The independent variables are nondimensionalized as $t = t^*/\bar{a}_{inj}^* R^*$, $z = z^*/R^*$, $r = r^*/R^*$.

The nondimensional governing equations for a given continuous flow region (ith region located between ith and ith$+1$ sources) can then be written as

$$\begin{aligned} &\partial \rho_i/\partial t + \nabla \cdot (\rho_i \mathbf{q}_i) = 0 \quad \text{(continuity)} \\ &\rho_i(Dq_i/Dt) + (1/\gamma)\nabla P_i = 0 \quad \text{(momentum)} \\ &D\sigma_i/Dt = 0 \quad \text{(entropy)} \\ &P_i = \rho_i T_i \quad \text{(state)} \\ &\mathbf{q}_i = \nabla \phi_i \quad \text{(irrotationality)} \end{aligned} \tag{1}$$

There are ξ sets of the previous equations, corresponding to ξ regions of uniform flow in the steady state.

The steady-state solution to the preceding equations require that all the $\bar{P}_i$, $\bar{u}_i$, etc., are constants and discontinuously change across each combustion source. The steady-state solutions are therefore represented as

$$\begin{aligned} \bar{m}_{\xi_i} &= \bar{\rho}_{i+1}\bar{u}_{i+1} - \bar{\rho}_i \bar{u}_i \\ \bar{P}_i &= \bar{u}_1(\bar{u}_1 - u_l) + 1 + \gamma \bar{\rho}_i \bar{u}_i (u_l - \bar{u}_i) \\ \bar{\rho}_i &= \bar{P}_i/\bar{T}_i \quad (i = 1, 2, \ldots, \xi) \end{aligned}$$

From an energy balance across the combustion sources the following is obtained

§ A stability study of a fully lined combustor with a low Mach number mean flow combustion distribution can be found in Ref. 10, pp. 139–146.

$$h_i^\circ = \text{const} \quad (i = 1, 2, \ldots, \xi)$$
$$\bar{T}_i = 1 + [(\gamma-1)/2](\bar{u}_1{}^2 - \bar{u}_i{}^2)$$

where $\bar{u}_1$ is the steady-state gas velocity at the injector face. ($\bar{u}_1$ is zero unless there is a planar source of combustion coincident with the injector face.)

The state variables, except entropy, are then expanded in a power series of some amplitude parameter (ε), i.e.,

$$\phi_i = \bar{\phi}_i + \varepsilon\phi_i' + 0(\varepsilon^2)$$
$$P_i = \bar{P}_i + \varepsilon P_i' + 0(\varepsilon^2)$$

Perturbations of entropy are neglected due to the small over-all effect they produce on the problem.[4]

The time dependent part of each perturbation is assumed to take the form $e^{i\omega t}$, $\omega = \alpha_R + i\alpha_I$ represents a complex frequency so that growth, decay, or neutral perturbation behavior is represented, respectively, by a negative, positive, or zero valued α_I.

With the preceding assumptions a first-order linearization is made which yields the following set of equations:

$$P_i' = (\gamma\bar{P}_i/\bar{\rho}_i)\rho_i' \tag{2}$$

$$\nabla^2\phi_i' + \omega^2\phi_i' = \frac{1}{\bar{T}_i}\left\{(\bar{T}_i - 1)\omega^2\phi_i' + 2i\omega\bar{u}_i\frac{\partial\phi_i'}{\partial z} + \bar{u}_i{}^2\frac{\partial^2\phi_i'}{\partial z^2}\right\} \tag{3}$$

$$P_i' = -\bar{\rho}_i\left\{\gamma i\omega\phi_i' + \gamma\bar{u}_i\frac{\partial\phi_i'}{\partial z}\right\} \tag{4}$$

A flow response between the pressure and the gas velocity is assumed to characterize the boundary conditions at the injector, nozzle and the acoustic liner.[7] These relationships measure the extent of energy addition or dissipation on the various surfaces. Mathematically, the boundary conditions are expressed as follows:

$$\left.\frac{\partial\phi_1'}{\partial z}\right|_{z=0} = \beta_{\text{inj}}\frac{\bar{a}_1}{\bar{P}_1}\left.P_1'\right|_{z=0}$$

$$\left.\frac{\partial\phi_i'}{\partial r}\right|_{r=1} = \beta_c\frac{\bar{a}_i}{\bar{P}_i}\left.P_i'\right|_{r=1} \quad (i = 1, 2, \ldots, \xi) \tag{5}$$

$$\left.\frac{\partial\phi_\xi'}{\partial z}\right|_{z=L/R} = \beta_{\text{noz}}\frac{\bar{a}_\xi}{\bar{P}_\xi}\left.P_\xi'\right|_{z=L/R}$$

β_{inj}, β_{noz}, β_c are, respectively, the injector, nozzle, and acoustic liner admittances, with $\beta_c = 0$ over the unlined portion of the chamber's cylindrical periphery.

The remaining $2(\xi - 1)$ boundary conditions necessary for the complete solution are obtained from the conservation of mass and momentum across the combustion sources. These "jump" relationships for the ith source located at ζ_i are as follows:

$$\rho_{i+1}u_{i+1} = \dot{m}_{\zeta i} + \rho_i u_i \quad \text{(continuity)}$$
$$P_{i+1} - P_i = \gamma\dot{m}_{\zeta i}u_l - \gamma\rho_{i+1}u_{i+1}^2 + \gamma\rho_i u_i{}^2 \quad \text{(momentum)}$$

where u_l = droplet velocity and $\dot{m}_{\zeta i}$ = mass generation flux for ith source.

A relationship for $\dot{m}'_{\zeta_i}$, the perturbed mass generation rate of the combustion sources, is then adopted by using the Crocco $n-\tau$ theory.[5] This model of combustion relates the mass flow perturbations to the pressure perturbations through the use of two parameters n, the interaction index, and τ, the time lag. This relationship takes the form

$$\dot{m}'_{\zeta_i} = N\bar{\dot{m}}_{\zeta_i}[P_i'/\bar{P}_i]_{\text{mean}}$$

where N = the combustion response = $n(1 - e^{-i\omega\tau})$

$$\left[\frac{P_i'}{\bar{P}_i}\right]_{\text{mean}} = -\gamma\frac{(\bar{\rho}_i + \bar{\rho}_{i+1})}{(\bar{P}_i + \bar{P}_{i+1})}\left.\left\{i\omega\phi' + \frac{(\bar{u}_i + \bar{u}_{i+1})}{2}\frac{\partial\phi'}{\partial z}\right\}\right|_{\zeta_i} \tag{6}$$

Using this combustion model a linearization of the "jump" relations produces the first-order matching conditions which take the following forms:

$$(\phi'_{i+1} - \phi_i)|_{\zeta_i} = [f_{3,i}(\phi', \bar{u}_i, \bar{u}_{i+1}) - N\hat{f}_{3,i}(\phi', \bar{u}_i, \bar{u}_{i+1})]|_{\zeta_i}$$

$$\left.\left(\frac{\partial\phi'_{i+1}}{\partial z} - \frac{\partial\phi_i'}{\partial z}\right)\right|_{\zeta_i} = [f_{4,i}(\phi', \bar{u}_i, \bar{u}_{i+1}) - N\hat{f}_{4,i}(\phi', \bar{u}_i, \bar{u}_{i+1})]|_{\zeta_i} \tag{7}$$

where

$$f_{3,i}(\phi', \bar{u}_i, \bar{u}_{i+1}) = \left.\left(C_{1,i}\phi' + C_{2,i}\frac{\partial\phi'}{\partial z}\right)\right|_{\zeta_i}$$

$$\hat{f}_{3,i}(\phi', \bar{u}_i, \bar{u}_{i+1}) = C_{3,i}\gamma\bar{\dot{m}}_{\zeta_i}\frac{(\bar{\rho}_i + \bar{\rho}_{i+1})}{(\bar{P}_i + \bar{P}_{i+1})} \times \left.\left[i\omega\phi' + \frac{(\bar{u}_i + \bar{u}_{i+1})}{2}\frac{\partial\phi'}{\partial z}\right]\right|_{\zeta_i}$$

$$f_{4,i}(\phi', \bar{u}_i, \bar{u}_{i+1}) = \left.\left(C_{4,i}\phi' + C_{5,i}\frac{\partial\phi'}{\partial z}\right)\right|_{\zeta_i}$$

$$\hat{f}_{4,i}(\phi', \bar{u}_i, \bar{u}_{i+1}) = \left.\frac{C_{6,i}}{C_{3,i}}\hat{f}_{3,i}(\phi', \bar{u}_i, \bar{u}_{i+1})\right|_{\zeta_i}$$

$C_{1,i}$, $C_{2,i}$, $C_{3,i}$, $C_{4,i}$, $C_{5,i}$, and $C_{6,i}$ are constants dependent upon the values of the steady-state variables of the zones adjacent to the ith combustion source.

The velocity potential (a Fourier expansion of the discontinuous collection of ϕ_i') is then found using a Green's function approach.

The Green's function, $G(\mathbf{R}/\mathbf{R}_0)$, is introduced into the problem through the following equation

$$\nabla^2 G(\mathbf{R}/\mathbf{R}_0) + \omega^2 G(\mathbf{R}/\mathbf{R}_0) = \delta(\mathbf{R} - \mathbf{R}_0) \quad \text{with} \quad \nabla G \cdot \mathbf{n} = 0$$

on the boundaries of the chamber. An eigenfunction expansion of $G(\mathbf{R}/\mathbf{R}_0)$ yields the following functional form

$$G(R/R_0) = \sum_{m=0}^{\infty}\sum_{l=1}^{\infty}\sum_{n=0}^{\infty}\frac{\Omega_{lmn}(\mathbf{R})\Omega_{lmn}(\mathbf{R}_0)}{(\omega^2 - \eta_{lmn}{}^2)}$$

where Ω_{lmn} is an orthonormal eigenfunction for the chamber

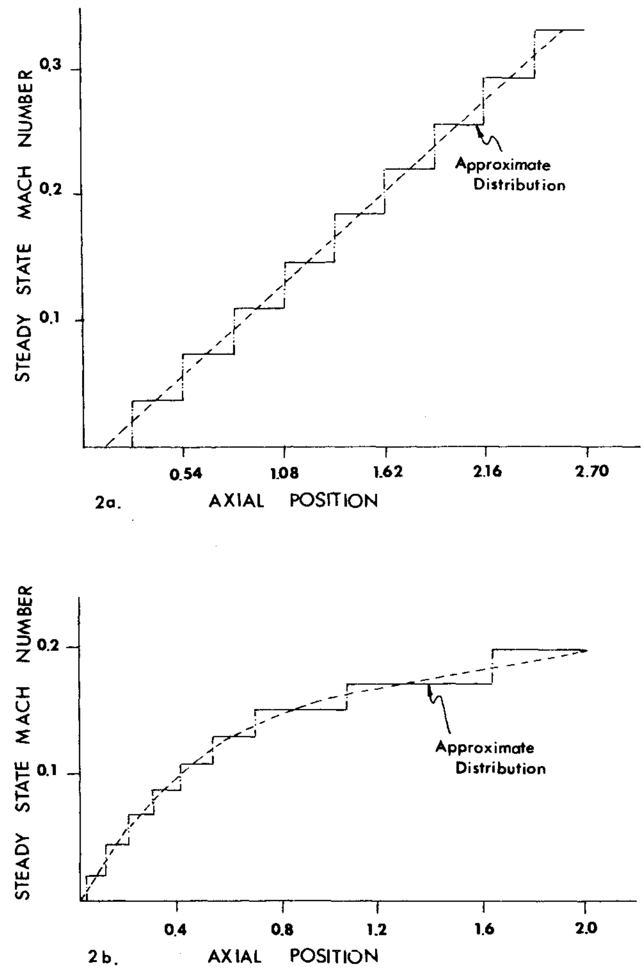

Fig. 2 Approximations of the steady-state Mach number variations for a) a "ramp" distribution; and b) a "Priem-Heidmann" distribution.

with no flow and homogeneous boundary conditions represented as

$$\Omega_{lmn}(\mathbf{R}) = (1/\Lambda_{lmn}^{1/2}) \cos m\theta J_m(\lambda_{lm} r) \cos n(\pi z/L)$$

$\Lambda_{lmn}^{1/2}$ is the normalization constant; η_{lmn} is the corresponding eigenvalue given as $\eta_{lmn}^2 = (n\pi/L)^2 + \lambda_{lm}^2$; λ_{lm} are the roots of $(d/dr)J_m(\lambda_{lm} r)|_{r=1} = 0$; and l, m, n are, respectively, the radial, azimuthal, and axial wave numbers.

Rewriting Eqs. (3) and (4) as

$$\begin{aligned} \nabla^2 \phi_i' + \omega^2 \phi_i' &= f_{1,i}(\phi_i', \bar{u}_i) \\ P_i' &= -f_{2,i}(\phi_i' \bar{u}_i) \end{aligned} \tag{8}$$

where $f_{1,i}$ and $f_{2,i}$ are R.H.S. of Eqs. (3) and (4), and introduction of Green's function described previously results in the following two integral equations. (Details of the development are presented in the Appendix.)

$$\begin{aligned} \phi' = \Omega_N + \sum_{i=1}^{\xi-1} \iint_{S_{\zeta_i}} \Big[& G_N(\mathbf{R}/\mathbf{R}_0)\{f_{3,i}(\phi', \bar{u}_i, \bar{u}_{i+1}) - \\ & N\hat{f}_{3,i}(\phi', \bar{u}_i, \bar{u}_{i+1})\} - \frac{\partial}{\partial z} G_N(\mathbf{R}/\mathbf{R}_0) \times \\ & \{f_{4,i}(\phi', \bar{u}_i, \bar{u}_{i+1}) - N\hat{f}_{4,i}(\phi', \bar{u}_i, \bar{u}_{i+1})\} \Big] dS_{\zeta_i} + \\ & \sum_{i=1}^{\xi} \iint_{S_{c_i}} \beta_c G_N(\mathbf{R}/\mathbf{R}_0) f_{2,i}(\phi', \bar{u}_i)\, dS_{c_i} + \\ & \iint_{S_{\text{noz}}} \beta_{\text{noz}} G_N(\mathbf{R}/\mathbf{R}_0) f_{2,\xi}(\phi', \bar{u}_\xi)\, dS_{\text{noz}} - \\ & \iint_{S_{\text{inj}}} \beta_{\text{inj}} G_N(\mathbf{R}/\mathbf{R}_0) f_{2,1}(\phi', \bar{u}_1)\, dS_{\text{inj}} + \\ & \sum_{i=1}^{\xi} \iiint_{V_i} G_N(\mathbf{R}/\mathbf{R}_0) f_{1,i}(\phi', \bar{u}_i)\, dV_i \end{aligned} \tag{9}$$

$$\begin{aligned} \omega^2 - \eta_{\hat{l}\hat{m}\hat{n}}^2 = \sum_{i=1}^{\xi-1} \iint_{S_{\zeta_i}} \Big[& \Omega_N \{f_{3,i}(\phi', \bar{u}_i, \bar{u}_{i+1}) - \\ & N\hat{f}_{3,i}(\phi, \bar{u}, \bar{u}_{i+1})\} - \frac{\partial \Omega_N}{\partial z}\{f_{4,i}(\phi', \bar{u}_i, \bar{u}_{i+1}) - \\ & N\hat{f}_{4,i}(\phi', \bar{u}_i, \bar{u}_{i+1})\} \Big] dS_{\zeta_i} + \sum_{i=1}^{\xi} \iint_{S_{c_i}} \beta_c \Omega_N f_{2,i}(\phi', \bar{u}_i)\, dS_{c_i} + \\ & \iint_{S_{\text{noz}}} \beta_{\text{noz}} \Omega_N f_{2,\xi}(\phi', \bar{u}_\xi)\, dS_{\text{noz}} - \\ & \iint_{S_{\text{inj}}} \beta_{\text{inj}} \Omega_N f_{2,1}(\phi', \bar{u}_1)\, dS_{\text{inj}} + \\ & \sum_{i=1}^{\xi} \iiint_{V_i} \Omega_N f_{1,i}(\phi', \bar{u}_i)\, dV_i \end{aligned} \tag{10}$$

where $G_N(\mathbf{R}/\mathbf{R}_0)$ is a modified Green's function given as

$$G_N(\mathbf{R}/\mathbf{R}_0) = \sum_{m=0}^{\infty} \sum_{l=1}^{\infty} \sum_{n=0}^{\infty} \frac{(1 - \delta_{m,\hat{m}} \delta_{l,\hat{l}} \delta_{n,\hat{n}}) \Omega_{lmn}(\mathbf{R}) \Omega_{lmn}(\mathbf{R}_0)}{\omega^2 - \eta_{lmn}^2}$$

Ω_N, a particular eigenfunction with wave number $\hat{l}$, $\hat{m}$, $\hat{n}$, physically represents a particular solution for the acoustic velocity potential in a closed wall right circular cylinder, and is presumed to be the acoustic mode closest in functional form to ϕ.

A successive approximation technique is employed to solve these integral equations for ϕ and N. The first approximation used is the separation of variables solution for the unlined chamber ($\beta_c = 0$). This first approximation takes the following form:

$$\tilde{\phi}_i' = (\alpha_i/\psi) \cos \hat{m}\theta J_{\hat{m}}(\lambda_{l\hat{m}} r) \{e^{iB_{1,i}Z} + C_i e^{iB_{2,i}Z}\} \tag{11}$$

$\{\alpha_i, C_i, \psi\}$ is a set of constants determined through the boundary conditions and the matching relationships. $B_{1,i}$, $B_{2,i}$, α_i, C_i are functions of ω, β_{noz} and $\tilde{N}$.

With the preceding first approximation, higher approximations for ϕ'^j and the corresponding N^j are obtained from the integral equations by using the last approximation obtained in order to evaluate the integrals of Eqs. (9) and (10). These higher approximations take the following form:

$$\phi'^j = \tilde{\phi}' + \cos \hat{m}\theta \sum_l \sum_n \mu_{ln}{}^j J_{\hat{m}}(\lambda_{l\hat{m}} r) \cos n \frac{\pi}{L} z \tag{12}$$

The successive approximation method for evaluation of N^j and $\mu_{ln}{}^j$ was coded into FORTRAN for evaluation on a CDC 6400 digital computer. Input to the computer program included a Mach number distribution, nozzle response, chamber length, liner response, liner length, and liner placement.

A 3×30 $\mu_{ln}{}^j$ matrix was used to approximate the doubly infinite Fourier-Bessel series for ϕ^j. This choice of matrix size was made on the basis of an error study in Ref. 9 and the comparison of series terms.

Results

Stability calculations were performed for two general combustion distributions. One distribution involved an even spatial distribution of combustion creating a linear variation of Mach number with axial length. This type of distribution is referred to as a "ramp" distribution (Fig. 2a). The second general distribution that was considered is a more realistic model which was obtained from a paper by Priem and Heidmann.[8] This distribution involves an increasing spread of combustion with axial distance from the injector (Fig. 2b).

The reference combustor chosen for this analysis had the following characteristics $L/R = 2.7$, $M_\xi = 0.33$, $G = 0.9166 + 0i$, $X = \frac{1}{3}L/R$, $K = 5.0 + 0i$, $\gamma = 1.2$, and $u_l = M_\xi/8.0$. This combustor was chosen so as to compare distributed combustion results with the concentrated combustion solutions of Refs. 9 and 7. Attention was restricted to oscillations of the first transverse mode type. The acoustic liner boundary condition was taken to be $\beta c = \text{const}$ (an increase in βc increases the effectiveness of the liner). This boundary condition is justified for oscillations with very small amplitudes (as dictated by a linear theory).[10] However, as the amplitude of the oscillations increase, βc becomes a function of the spatial variables through the amplitude. This increases the complexity of the mathematics of the problem, although it can be treated by the integral method. It would seem more appropriate to include nonlinear liner effects in a complete nonlinear treatment of the problem.

Results in this study are plotted on the $n-\tau$ plane. n and τ are related to N by the expression $N = n(1 - e^{-i\omega\tau})$. ω_0 is a reference frequency defined as the fundamental frequency of the first transverse mode ($\omega_0 = 1.841184$). τ, the time lag, is nondimensionalized as $\tau = \tau^* a^*/R^*$.

On the $n-\tau$ plane combustion responses which produce $n-\tau$ values that are above the parabola-shaped neutral stability curve produce linear growth of the oscillations and below the curve produce linear decay. The critical point on this curve is the minimum n. Values of n which are below $n_{\min}$ will produce decaying oscillations regardless of the value of τ.

On Fig. 3, the results of stability calculations which were made for a "ramp" flow distribution are compared with the concentrated combustion solutions of Ref. 9. It can be seen from this figure that distributing the combustion has a marked stabilizing effect even for an unlined chamber. Also the effectiveness of the liner is enhanced by spreading out the combustion sources. Loops which are a characteristic feature of concentrated combustion solutions do not appear in the distributed combustion results (loops indicate combined longitudinal mode behavior). Consequently, combined longitudinal mode behavior is less likely in the distributed combustion case.

A comparison of stability results between the "Priem-Heidmann" distribution and a "ramp" distribution with the same exit Mach number for unlined and $\frac{1}{3}L/R$ lined combustors is shown in Fig. 4. The curves shift to the left on the $n-\tau$ plane for the "Priem-Heidmann" distribution relative to the "ramp." However, the minimum n for both cases remains virtually

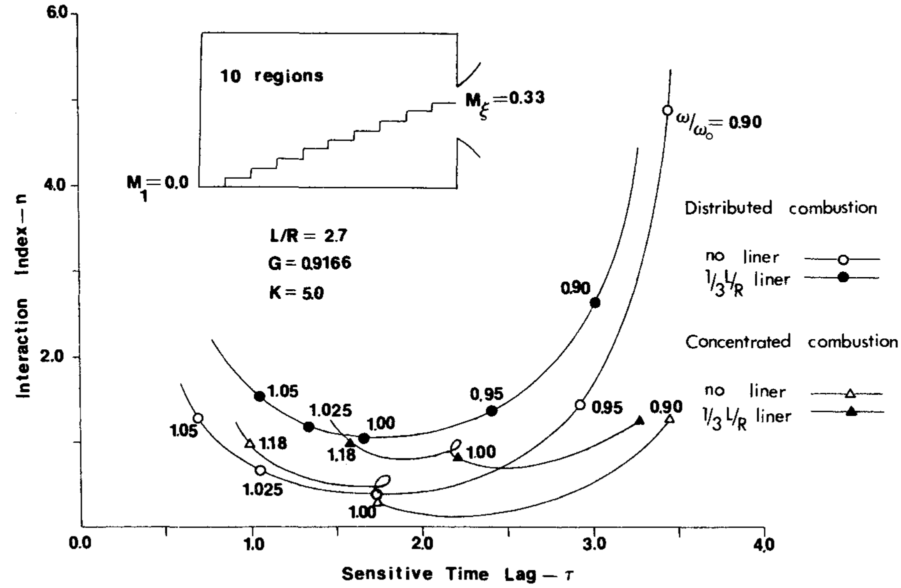

Fig. 3 A comparison of stability results between a concentrated combustion model[9] and a "ramp" distribution combustion model for unlined and $\frac{1}{3}L/R$ lined combustors.

the same. This indicates that the exact shape of the Mach number distribution has relatively little importance as far as the minimum combustion response necessary for instability is concerned.

One difference in the stability behavior appears between the "ramp" and the "Priem-Heidmann" distribution. The effect of placement of liner is more significant when the "Priem-Heidmann" distribution is used. Figure 5 shows this effect. As X_1 increases (the liner is moved farther from the injector) stability decreases. The stability shift is small but is clearly evident. In the "Priem-Heidmann" distribution the driving abilities of the combustion sources decrease with axial position from the injector. Thus liner effectiveness is greatest when the liner is placed with the maximum of driving capacity of the combustion (location of greatest Mach number gradient).

Conclusions

A stability-oriented integral method for flow with combustion source discontinuities has been developed and applied to evaluate the effects of partial length acoustic liners on liquid propellant combustors. Results of this application imply that an axial spread of combustion has a significant stabilizing effect. Instabilities in this type of flow tend to be of a pure transverse type with little possibility of combined longitudinal mode behavior.

A comparison between two types of combustion source distributions indicate that over-all stability is slightly affected by the positioning of the combustion sources. However, the acoustic liner is most effective when placed at the location of the greatest Mach number gradient.

Finally, the integral method developed in this paper is not limited to the combustion instability problem treated here and should be applicable to other problems in which confined or semi-confined flows with sources occur.

Appendix

General Integral Equation

A set of partial differential equations of the type

$$\nabla^2\phi_i + \omega^2\phi_i = f(\phi_i, r, \theta, z)$$

is transformed into a single integral equation with the use of a Green's function that is continuous throughout the whole region

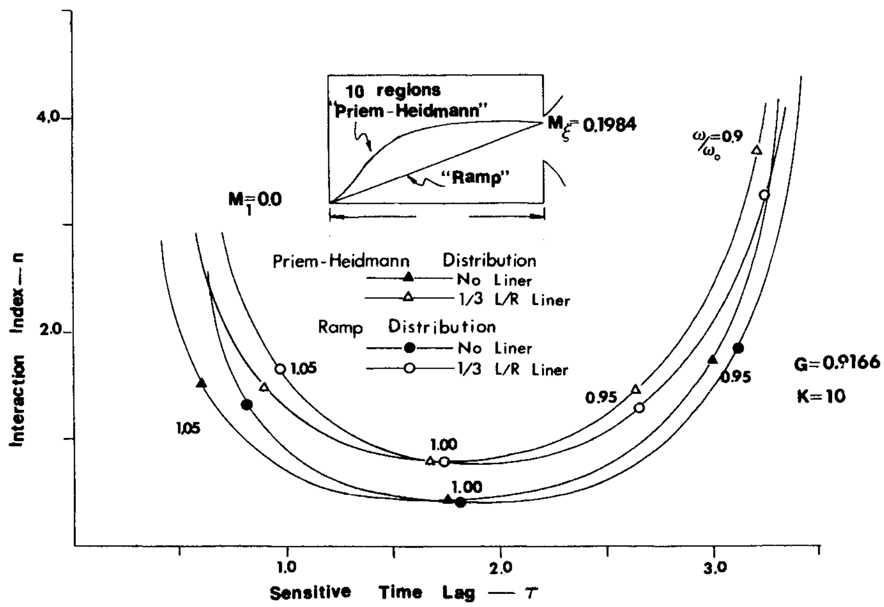

Fig. 4 Comparison of stability results for an unlined and a $\frac{1}{3}L/R$ lined combustor using the "Priem-Heidmann" distribution and a "ramp" distribution with the same exit Mach number.

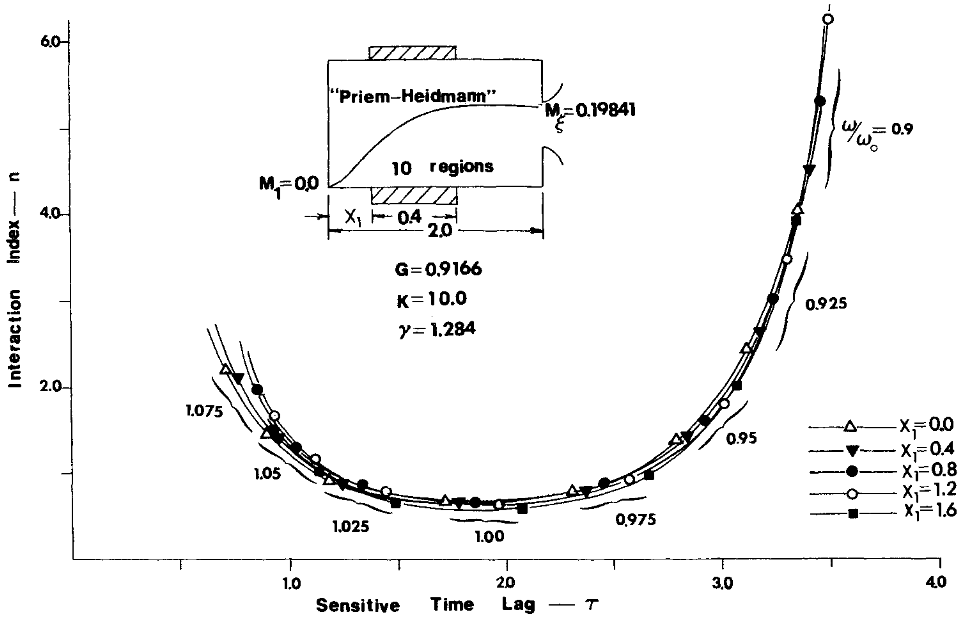

Fig. 5 Effect of a placement of $\frac{1}{3}L/R$ liner on a combustor with the "Priem-Heidmann" distribution.

(the total collection of the connected regions to which the ϕ_i's apply). The standard technique of integral transformation is made for each member, ϕ_i, of the potential differential set with integrations over the volume and surfaces to which ϕ_i applies. Summing the resulting set of integrals yields the following general form:

$$\sum_{i=1}^{\xi}\iiint_{V_i}\phi_i\,\delta(\mathbf{R}-\mathbf{R}_0)\,dV_i =$$

$$\sum_{i=1}^{\xi-1}\iint_{S_{\zeta_i}}\left\{G(\mathbf{R}/\mathbf{R}_0)\left(\frac{\partial\phi_{i+1}}{\partial z}-\frac{\partial\phi_i}{\partial z}\right)\bigg|_{z_{\zeta_i}} - \frac{\partial G(R/R_0)}{\partial z}(\phi_{i+1}-\phi_i)\bigg|_{z_{\zeta_i}}\right\}dS_{\zeta_i} +$$

$$\sum_{i=1}^{\xi}\iiint_{V_i}G(\mathbf{R}/\mathbf{R}_0)f(\phi_i,r,\theta,z)\,dV_i -$$

$$\iint_{S_{\text{noz}}}G(\mathbf{R}/\mathbf{R}_0)\frac{\partial\phi_\xi}{\partial z}\bigg|_{Z=L/R}dS_{\text{noz}} +$$

$$\iint_{S_{\text{inj}}}G(\mathbf{R}/\mathbf{R}_0)\frac{\partial\phi_1}{\partial z}\bigg|_{Z=0}dS_{\text{inj}} -$$

$$\sum_{i=1}^{\xi}\iint_{S_{C_i}}G(\mathbf{R}/\mathbf{R}_0)\frac{\partial\phi_i}{\partial r}\bigg|_{r=1}dS_{C_i}$$

where V_i is the volume of the ith continuous flow region; S_{ζ_i} is the surface of the planar combustion source which interconnects the ith and the ith + 1 regions; S_{inj} is the injector surface area; S_{noz} is the nozzle surface area; and S_{c_i} is the peripheral surface area within region i.

Because of the eigenfunction expansion of $G(\mathbf{R}/\mathbf{R}_0)$ the right-hand integral represents a Fourier-Bessel expansion of the discontinuous collection of ϕ_i'. The first integral on the left-hand side involves two boundary conditions at the combustion sources which requires a matching of the potentials and their derivatives. The last three integrals involve nozzle, injector, and liner boundary conditions in Eq. (5). (The injector boundary condition is a function of the combustion response.)

A special treatment of the integral equation can be made if the integrals are of a homogeneous type as was the case in this application of the method. Owing to the form of the Green's function, the potential ϕ will have an eigenfunction expansion. One term of this series, Ω_N, will dominate the character of the solution. Since the amplitude of ϕ remains undefined we can set

$$\sum_{i=1}^{\xi}\iiint_{V_i}\phi_i\,\Omega_N\,dV_i = 1$$

This condition allows a separation of the main integral equation into two similar equations—one equation for the wave form of ϕ and another equation for the corresponding eigenvalue of the system.

References

[1] Crocco, L., Grey, J., and Harrje, D. T., "Theory of Liquid Propellant Rocket Combustion Instability and its Experimental Verification," *American Rocket Society Journal*, Vol. 30, No. 2, Feb. 1960, pp. 150–168.

[2] Crocco, L. and Mitchell, C. E., "Nonlinear Periodic Oscillations in Rocket Motors with Distributed Combustion," *Combustion Science and Technology*, Vol. 1, 1969, pp. 147–169.

[3] Crocco, L. and Sirignano, W. A., "Effect of the Transverse Velocity Component on the Nonlinear Behavior of Short Nozzles," *AIAA Journal*, Vol. 4, No. 8, Aug. 1966, pp. 1428–1430.

[4] Mitchell, C. E., "The Effect of Entropy Waves on High Frequency Pressure Oscillations in Liquid Rocket Motors," *Combustion Science and Technology*, Vol. 1, 1970, pp. 269–274.

[5] Crocco, L. and Cheng, S. I., *Theory of Combustion Instability in Liquid Propellant Rocket Motors*, AGARDograph No. 8, Butterworths, London, 1956, pp. 19–24.

[6] Oberg, C. L. and Kuluva, N. M., "Acoustic Liners for Large Engines," Final Report, Contract 21345, 1969, NASA.

[7] Priem, R. J. and Rice, E. J., "Combustion Instability with Finite Mach Number Flow and Acoustic Liners," TMX-52412, 1968, NASA.

[8] Priem, R. J. and Heidmann, M. R., "Propellant Vaporization as a Design Criteria for Rocket Engine Combustion Chambers," TR R-67, 1960, NASA.

[9] Mitchell, C. E., Espander, W. R., and Baer, M. R., "A Study of the Effects of Partial Length Acoustic Liners in Liquid Propellant Rocket Motors," 8th JANAF Combustion Meeting, CPIA Pub. 220, Nov. 1971, pp. 795–804.

[10] Zinn, B. T., *Liquid Propellant Rocket Combustion Instability*, edited by D. T. Harrje and F. H. Reardon, NASA SP-194, 1972, Chap. 8.3.1, pp. 400–410.